5th International Workshop on
Dark Matter, Dark Energy and Matter-Antimatter Asymmetry

Minimal Gauged $\mathrm{U}(1)_{L_\alpha - L_\beta}$ Models Driven into a Corner

in collaboration with K. Asai, K. Hamaguchi, N. Nagata, K. Tsumura

arXiv:1811.0757

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- Minimal models
- Neutrino mass structure
- Summary
- Leptogenesis

Introduction

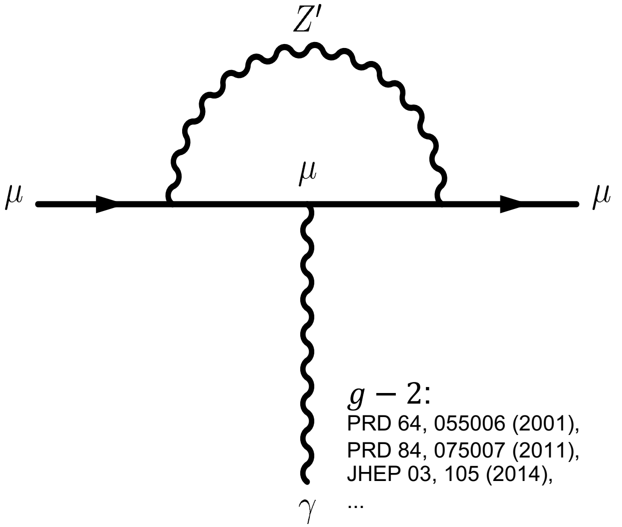
Introduction

- Possible extension of SM

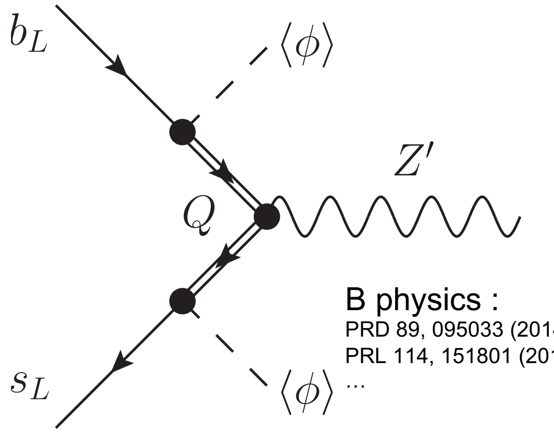
$$\underbrace{\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y}_{\mathrm{G_{SM}}}\times\mathrm{U}(1)_{L_\alpha-L_\beta} \quad \alpha,\beta\in e,\mu,\tau$$

Mod.Phys.Lett. A6 (1991) 527-530
Phys. Rev. D 43, R22
…

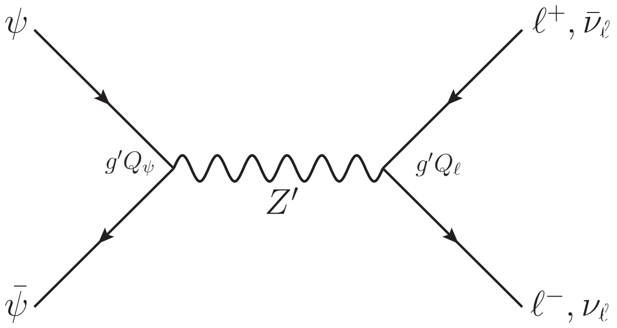
- $\mathrm{U}(1)_{L_\mu-L_\tau}$



$g-2$:
PRD 64, 055006 (2001),
PRD 84, 075007 (2011),
JHEP 03, 105 (2014),
...



B physics :
PRD 89, 095033 (2014),
PRL 114, 151801 (2015),
...



DM :
PLB 752 59–65 (2016,
PLB 756, 1–5 (2016),
JHEP 02, 123 (2017),
…

Introduction

- Light neutrino mass → adding RH neutrinos

$$M_\nu = -M_D M_R^{-1} M_D^T$$

Seesaw mechanism

Phys. Lett. B67 (1977) 421–428

Conf. Proc. C7902131 (1979) 95–99

…

mass terms tightly restricted by the $\mathrm{U}(1)_{L_\alpha - L_\beta}$ symmetry

$$\begin{pmatrix} * & 0 & 0 \\ 0 & 0 & * \\ 0 & * & 0 \end{pmatrix} \simeq \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix} \begin{pmatrix} * & 0 & 0 \\ 0 & 0 & * \\ 0 & * & 0 \end{pmatrix} \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$$

block-diagonal

this simple structure fails to explain the sizable neutrino mixing…

Introduction

- Spontaneous breaking of $\mathrm{U}(1)_{L_\alpha - L_\beta}$ symmetry

  introducing additional scalar fields

$$M_\nu^{-1} \simeq \begin{pmatrix} * & * & * \\ * & \boxed{0} & * \\ * & * & \boxed{0} \end{pmatrix}$$

or

$$M_\nu \simeq \begin{pmatrix} * & \boxed{0} & * \\ 0 & \boxed{0} & * \\ * & * & * \end{pmatrix}$$

Two-zero conditions lead to strong predictive power!

$$\sum_i m_i\,, \delta\,, \alpha_2\,, \alpha_3, \ldots$$

Minimal models

Model 1 : SM + 3 RH neutrinos + 1 scalar singlet

Model 2 : SM + 3 RH neutrinos + 1 scalar doublet

Model 1

- Setup
  SM
  3 RH neutrinos
  scalar singlet σ
- Charge assignment

fields	$\mathrm{U}(1)_{L_\mu - L_\tau}$
L_e, e_R, N_e	0
L_μ, μ_R, N_μ	+1
L_τ, τ_R, N_τ	-1
σ	+1

$$\begin{aligned}\Delta\mathcal{L} =& - y_e e_R^c L_e H^\dagger - y_\mu \mu_R^c L_\mu H^\dagger - y_\tau \tau_R^c L_\tau H^\dagger \\ & - \lambda_e N_e^c (L_e \cdot H) - \lambda_\mu N_\mu^c (L_\mu \cdot H) - \lambda_\tau N_\tau^c (L_\tau \cdot H) \\ & - \frac{1}{2} M_{ee} N_e^c N_e^c - M_{\mu\tau} N_\mu^c N_\tau^c - \lambda_{e\mu} \sigma N_e^c N_\mu^c - \lambda_{e\tau} \sigma^* N_e^c N_\tau^c + \text{h.c.}\end{aligned}$$

$$\mathcal{L}_{\text{mass}}^{(N)} = -(\nu_e, \nu_\mu, \nu_\tau) M_D \begin{pmatrix} N_e^c \\ N_\mu^c \\ N_\tau^c \end{pmatrix} - \frac{1}{2} (N_e^c, N_\mu^c, N_\tau^c) M_R \begin{pmatrix} N_e^c \\ N_\mu^c \\ N_\tau^c \end{pmatrix} + \text{h.c.}$$

$$M_D = \frac{v}{\sqrt{2}} \begin{pmatrix} \lambda_e & 0 & 0 \\ 0 & \lambda_\mu & 0 \\ 0 & 0 & \lambda_\tau \end{pmatrix} \qquad M_R = \begin{pmatrix} M_{ee} & \lambda_{e\mu}\langle\sigma\rangle & \lambda_{e\tau}\langle\sigma\rangle \\ \lambda_{e\mu}\langle\sigma\rangle & 0 & M_{\mu\tau} \\ \lambda_{e\tau}\langle\sigma\rangle & M_{\mu\tau} & 0 \end{pmatrix}$$

$$\mathcal{L}_{\text{mass}}^{(L)} = -(e_L, \mu_L, \tau_L) M_\ell \begin{pmatrix} e_R^c \\ \mu_R^c \\ \tau_R^c \end{pmatrix} + \text{h.c.} \qquad M_\ell = \frac{v}{\sqrt{2}} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

Model 2

- Setup
  SM
  3 RH neutrinos
  scalar doublet Φ_1

$\mathrm{U}(1)_{L_\mu - L_\tau}(\Phi_1) = +1$

$$\begin{aligned}\Delta\mathcal{L} = &- y_e e_R^c L_e \Phi_2^\dagger - y_\mu \mu_R^c L_\mu \Phi_2^\dagger - y_\tau \tau_R^c L_\tau \Phi_2^\dagger - y_{\mu e} e_R^c L_\mu \Phi_1^\dagger - y_{e\tau} \tau_R^c L_e \Phi_1^\dagger \\ &- \lambda_e N_e^c (L_e \cdot \Phi_2) - \lambda_\mu N_\mu^c (L_\mu \cdot \Phi_2) - \lambda_\tau N_\tau^c (L_\tau \cdot \Phi_2) \\ &- \lambda_{\tau e} N_e^c (L_\tau \cdot \Phi_1) - \lambda_{e\mu} N_\mu^c (L_e \cdot \Phi_1) - \frac{1}{2} M_{ee} N_e^c N_e^c - M_{\mu\tau} N_\mu^c N_\tau^c + \text{h.c.}\end{aligned}$$

$$M_D = \frac{1}{\sqrt{2}}\begin{pmatrix} \lambda_e v_2 & \lambda_{e\mu} v_1 & 0 \\ 0 & \lambda_\mu v_2 & 0 \\ \lambda_{\tau e} v_1 & 0 & \lambda_\tau v_2 \end{pmatrix} \quad M_R = \begin{pmatrix} M_{ee} & 0 & 0 \\ 0 & 0 & M_{\mu\tau} \\ 0 & M_{\mu\tau} & 0 \end{pmatrix} \quad M_\ell = \frac{1}{\sqrt{2}}\begin{pmatrix} y_e v_2 & 0 & y_{e\tau} v_1 \\ y_{\mu e} v_1 & y_\mu v_2 & 0 \\ 0 & 0 & y_\tau v_2 \end{pmatrix}$$

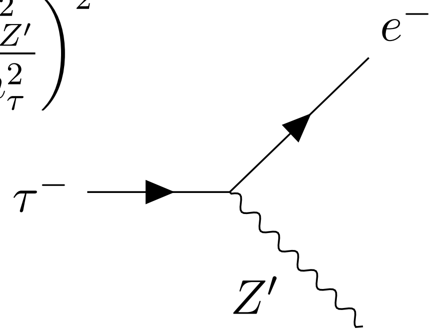
$\mathrm{U}(1)_{L_\mu - L_\tau}(\Phi_1) = -1$

$$\begin{aligned}\Delta\mathcal{L} = &- y_e e_R^c L_e \Phi_2^\dagger - y_\mu \mu_R^c L_\mu \Phi_2^\dagger - y_\tau \tau_R^c L_\tau \Phi_2^\dagger - y_{\tau e} e_R^c L_\tau \Phi_1^\dagger - y_{e\mu} \mu_R^c L_e \Phi_1^\dagger \\ &- \lambda_e N_e^c (L_e \cdot \Phi_2) - \lambda_\mu N_\mu^c (L_\mu \cdot \Phi_2) - \lambda_\tau N_\tau^c (L_\tau \cdot \Phi_2) \\ &- \lambda_{\mu e} N_e^c (L_\mu \cdot \Phi_1) - \lambda_{e\tau} N_\tau^c (L_e \cdot \Phi_1) - \frac{1}{2} M_{ee} N_e^c N_e^c - M_{\mu\tau} N_\mu^c N_\tau^c + \text{h.c.}\end{aligned}$$

$$M_D = \frac{1}{\sqrt{2}}\begin{pmatrix} \lambda_e v_2 & 0 & \lambda_{e\tau} v_1 \\ \lambda_{\mu e} v_1 & \lambda_\mu v_2 & 0 \\ 0 & 0 & \lambda_\tau v_2 \end{pmatrix} \quad M_R = \begin{pmatrix} M_{ee} & 0 & 0 \\ 0 & 0 & M_{\mu\tau} \\ 0 & M_{\mu\tau} & 0 \end{pmatrix} \quad M_\ell = \frac{1}{\sqrt{2}}\begin{pmatrix} y_e v_2 & y_{e\mu} v_1 & 0 \\ 0 & y_\mu v_2 & 0 \\ y_{\tau e} v_1 & 0 & y_\tau v_2 \end{pmatrix}$$

LFV process in Model 2

$$\Gamma(\tau \to eZ') = \frac{g_{Z'}^2 m_\tau}{128\pi} \sin^2 2\theta_L \left(2 + \frac{m_\tau^2}{m_{Z'}^2}\right)\left(1 - \frac{m_{Z'}^2}{m_\tau^2}\right)^2$$



$$M_\ell = U_L^* \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} U_R^T$$

$\sin 2\theta_L \simeq 0$

$\theta_L \simeq 0$ or $\underbrace{\pi/2}_{\text{e.g.}}$

$$U_L \simeq \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \Rightarrow Q_{\mu-\tau} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \to \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = Q_{\mu-e}$$

$\mathrm{U}(1)_{L_\mu - L_\tau}$ $\cancel{\mathrm{U}(1)_{L_e - L_\tau}}$ $\cancel{\mathrm{U}(1)_{L_e - L_\mu}}$

$m_{Z'}$ below EW scale
ruled out by experiments

JHEP (2018) 2018: 53
JHEP (2018) 2018: 94
PRL **113**, 091801
...

Neutrino mass structure

Analysis of M_ν : Model 1

$$M_D = \frac{v}{\sqrt{2}}\begin{pmatrix} \lambda_e & 0 & 0 \\ 0 & \lambda_\mu & 0 \\ 0 & 0 & \lambda_\tau \end{pmatrix} \qquad M_R = \begin{pmatrix} M_{ee} & \lambda_{e\mu}\langle\sigma\rangle & \lambda_{e\tau}\langle\sigma\rangle \\ \lambda_{e\mu}\langle\sigma\rangle & 0 & M_{\mu\tau} \\ \lambda_{e\tau}\langle\sigma\rangle & M_{\mu\tau} & 0 \end{pmatrix}$$

m_1, m_2, m_3, $\sum_i m_i$

$$f(\theta_{12}, \theta_{23}, \theta_{13}, \delta m^2, \Delta m^2)$$

δ, α_2, α_3, $\langle m_{\beta\beta}\rangle$

$$\left[U_L U_{\mathrm{PMNS}} (M_\nu^{\mathrm{d}})^{-1} U_{\mathrm{PMNS}}^T U_L^T\right]_{\mu\mu} = 0$$
$$\left[U_L U_{\mathrm{PMNS}} (M_\nu^{\mathrm{d}})^{-1} U_{\mathrm{PMNS}}^T U_L^T\right]_{\tau\tau} = 0$$

solve

match

$$\delta m^2 = \Delta m^2_{\mathrm{sol}},$$
$$\Delta m^2 = \Delta m^2_{\mathrm{atm}} - \frac{1}{2}\Delta m^2_{\mathrm{sol}}$$

$$M_\nu = -M_D M_R^{-1} M_D^T$$
$$M_\nu = U_\nu^* M_\nu^{\mathrm{d}} U_\nu^\dagger$$
$$U_{\mathrm{PMNS}} \equiv U_L^\dagger U_\nu$$

$$M_\nu^{-1} = -(M_D^{-1})^T M_R M_D^{-1} = \begin{pmatrix} * & * & * \\ * & \boxed{0} & * \\ * & * & \boxed{0} \end{pmatrix} = U_\nu (M_\nu^{\mathrm{d}})^{-1} U_\nu^T$$

two-zero minor

Analysis of M_ν : Model 2 with $\mathrm{U}(1)_{L_\mu - L_\tau}(\Phi_1) = +1$

$$M_D = \frac{1}{\sqrt{2}}\begin{pmatrix} \lambda_e v_2 & \lambda_{e\mu} v_1 & 0 \\ 0 & \lambda_\mu v_2 & 0 \\ \lambda_{\tau e} v_1 & 0 & \lambda_\tau v_2 \end{pmatrix} \qquad M_R = \begin{pmatrix} M_{ee} & 0 & 0 \\ 0 & 0 & M_{\mu\tau} \\ 0 & M_{\mu\tau} & 0 \end{pmatrix}$$

$$M_\nu = -\begin{pmatrix} \frac{\lambda_e^2 v_2^2}{2M_{ee}} & 0 & \frac{\lambda_{e\mu}\lambda_\tau v_1 v_2}{2M_{\mu\tau}} + \frac{\lambda_e \lambda_{\tau e} v_1 v_2}{2M_{ee}} \\ 0 & 0 & \frac{\lambda_\mu \lambda_\tau v_2^2}{2M_{\mu\tau}} \\ \frac{\lambda_{e\mu}\lambda_\tau v_1 v_2}{2M_{\mu\tau}} + \frac{\lambda_e \lambda_{\tau e} v_1 v_2}{2M_{ee}} & \frac{\lambda_\mu \lambda_\tau v_2^2}{2M_{\mu\tau}} & \frac{\lambda_{\tau e}^2 v_1^2}{2M_{ee}} \end{pmatrix}$$

$$\left[U_L^* U_{PMNS}^* M_\nu^{\mathrm{d}} U_{\mathrm{PMNS}}^\dagger U_L^\dagger\right]_{e\mu} = 0$$
$$\left[U_L^* U_{PMNS}^* M_\nu^{\mathrm{d}} U_{\mathrm{PMNS}}^\dagger U_L^\dagger\right]_{\mu\mu} = 0$$

Two-zero texture

$$M_\nu = \begin{pmatrix} * & \boxed{0} & * \\ 0 & \boxed{0} & * \\ * & * & * \end{pmatrix}$$

Predictions of $\sum_i m_i$, δ , α_2 , α_3…

$$f(\theta_{12}, \theta_{23}, \theta_{13}, \delta m^2, \Delta m^2)$$

Input parameters

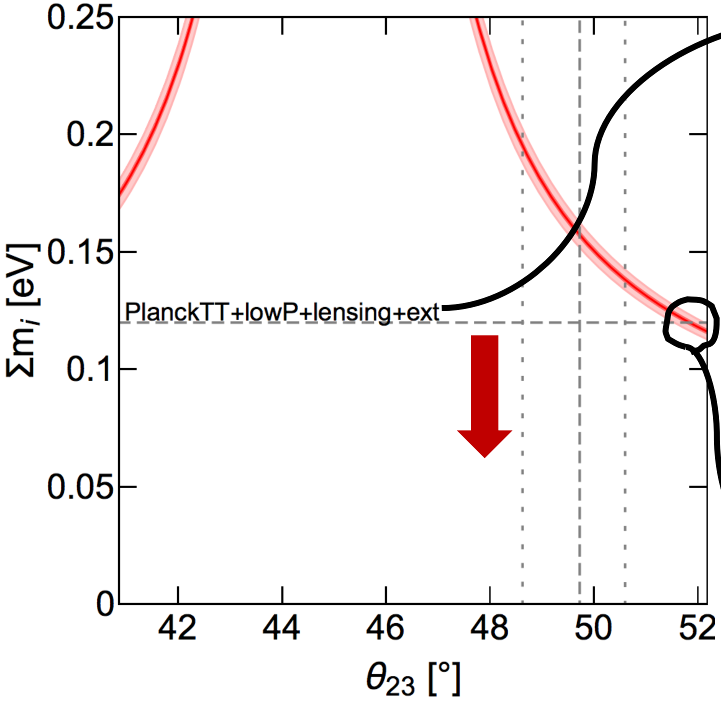
global-fitting group

NuFIT 4.0 (2018)

with SK atmospheric data		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 9.3$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
	$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$
	$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.75}$	$31.62 \to 36.27$
	$\sin^2\theta_{23}$	$0.582^{+0.015}_{-0.019}$	$0.428 \to 0.624$	$0.582^{+0.015}_{-0.018}$	$0.433 \to 0.623$
	$\theta_{23}/^\circ$	$49.7^{+0.9}_{-1.1}$	$40.9 \to 52.2$	$49.7^{+0.9}_{-1.0}$	$41.2 \to 52.1$
	$\sin^2\theta_{13}$	$0.02240^{+0.00065}_{-0.00066}$	$0.02044 \to 0.02437$	$0.02263^{+0.00065}_{-0.00066}$	$0.02067 \to 0.02461$
	$\theta_{13}/^\circ$	$8.61^{+0.12}_{-0.13}$	$8.22 \to 8.98$	$8.65^{+0.12}_{-0.13}$	$8.27 \to 9.03$
	$\delta_{\mathrm{CP}}/^\circ$	217^{+40}_{-28}	$135 \to 366$	280^{+25}_{-28}	$196 \to 351$
	$\dfrac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$
	$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.525^{+0.033}_{-0.031}$	$+2.431 \to +2.622$	$-2.512^{+0.034}_{-0.031}$	$-2.606 \to -2.413$

Neutrino phenomenology : Model 1

$U(1)_{L_\mu - L_\tau}$ Normal ordering :

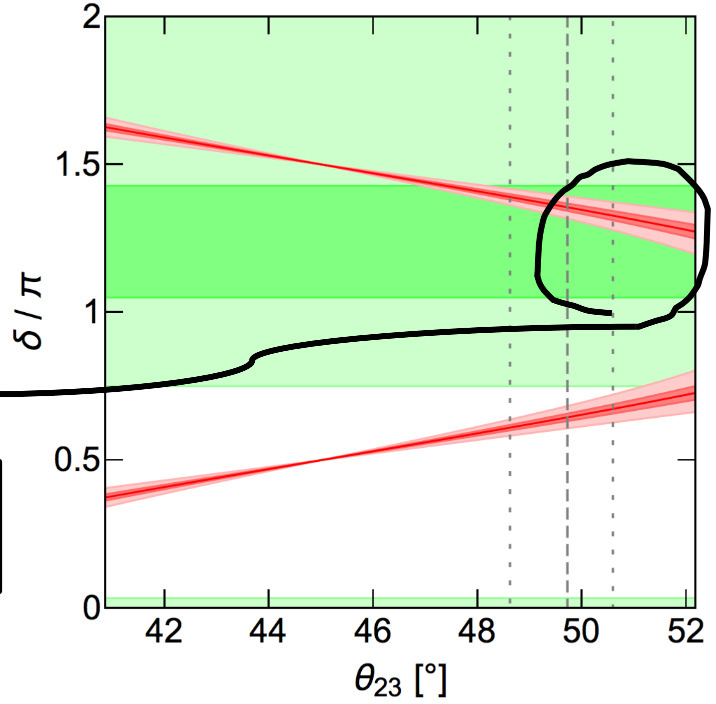


The tight bound set by Planck exp.

$\sum_i m_i < 0.12$ eV

Planck, arXiv:1807.06209

the region favored by experiments



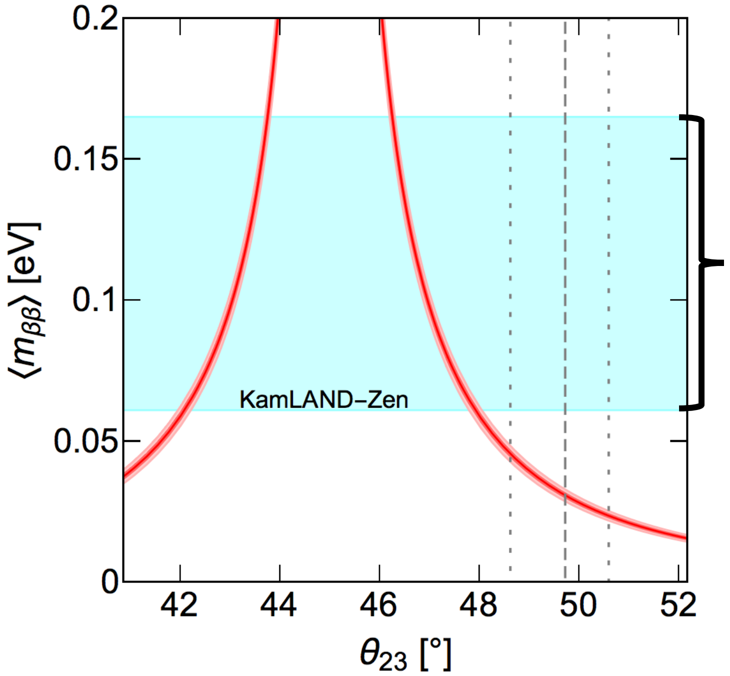
$U(1)_{L_\mu - L_\tau}$ Inverted ordering

$U(1)_{L_e - L_\mu}$, $U(1)_{L_e - L_\tau}$

: incorrect mass ordering, no real δ_{CP}

Neutrino phenomenology : Model 1

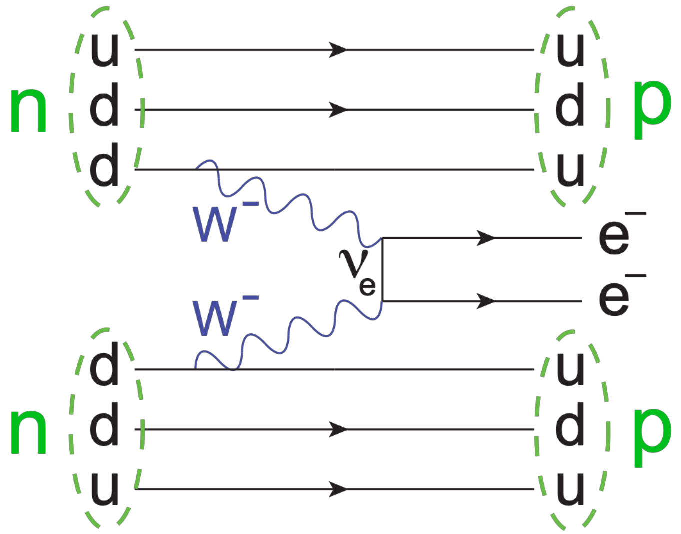
$U(1)_{L_\mu - L_\tau}$ Normal ordering :



The strongest bound set by KamLAND-Zen exp.

$\langle m_{\beta\beta} \rangle < 61\sim165$ meV

PRL 117, 082503 (2016)

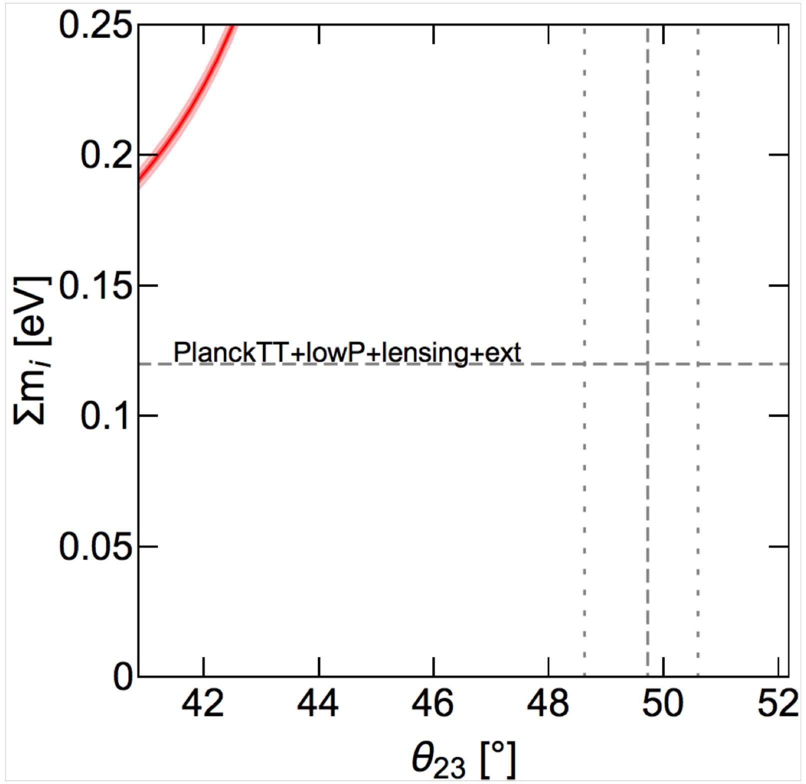


$$\Gamma \propto G_F^4 \times \langle m_{\beta\beta} \rangle^2 \times |M_{\mathrm{nucl}}|^2$$

$$\langle m_{\beta\beta} \rangle \equiv \left| \sum_i (U_{\mathrm{PMNS}})_{ei}^2 m_i \right| = \left| c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{i\alpha_2} m_2 + s_{13}^2 e^{i(\alpha_3 - 2\delta)} m_3 \right|$$

Neutrino phenomenology : Model 2

$U(1)_{L_\mu - L_\tau}$ Normal ordering with $\mathrm{U}(1)_{L_\mu - L_\tau}(\Phi_1) = +1$:



The tight bound set by Planck exp.

$$\sum_i m_i < 0.12 \text{ eV}$$

Planck, arXiv:1807.06209

all $\mathrm{U}(1)_{L_\alpha - L_\beta}$ models with two scalar doublets are excluded

Summary

Summary

- Minimal $\mathrm{U}(1)_{L_\alpha - L_\beta}$ models are analyzed in this work
- Only one survives experimental constraints

  $\mathrm{U}(1)_{L_\mu - L_\tau}$ extension with an extra scalar singlet

  - Normal ordering

  - $\sum_i m_i \gtrsim 0.12$ eV

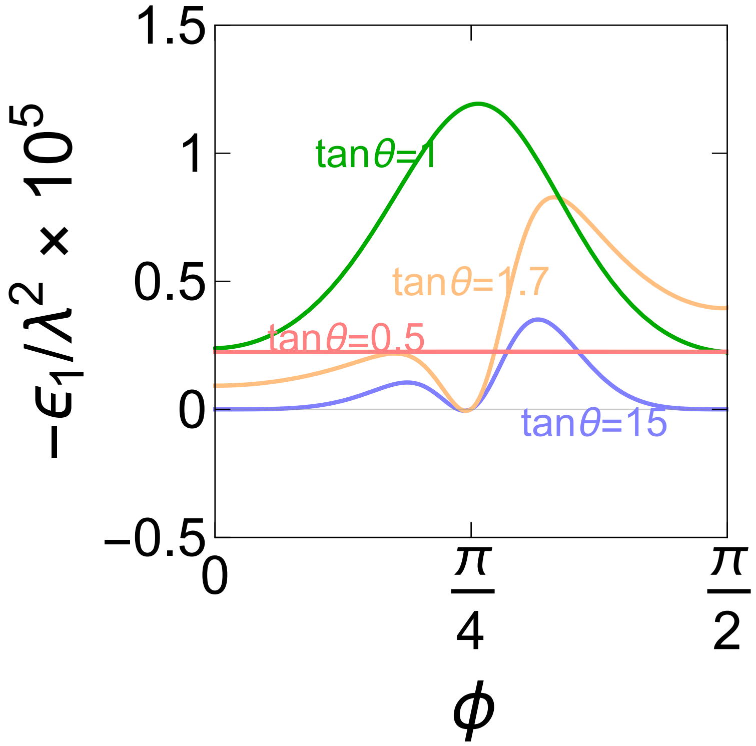
  - $\theta_{23} \simeq 52°$

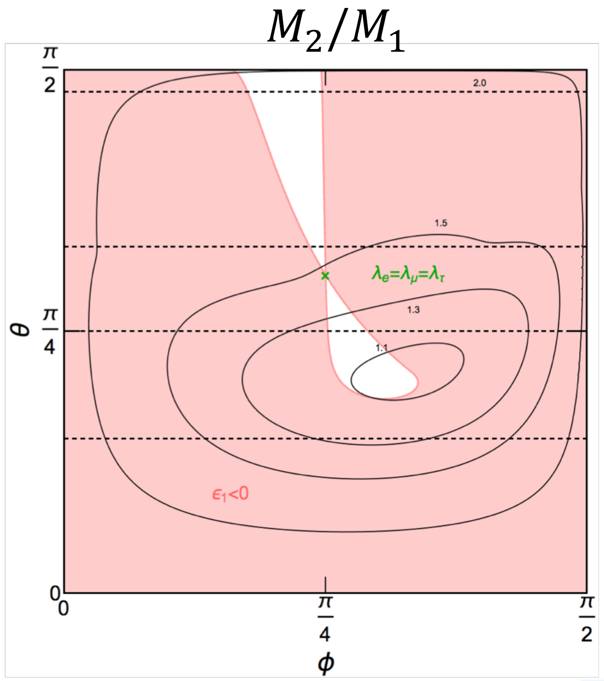
  - $\langle m_{\beta\beta} \rangle \gtrsim 0.016$ eV

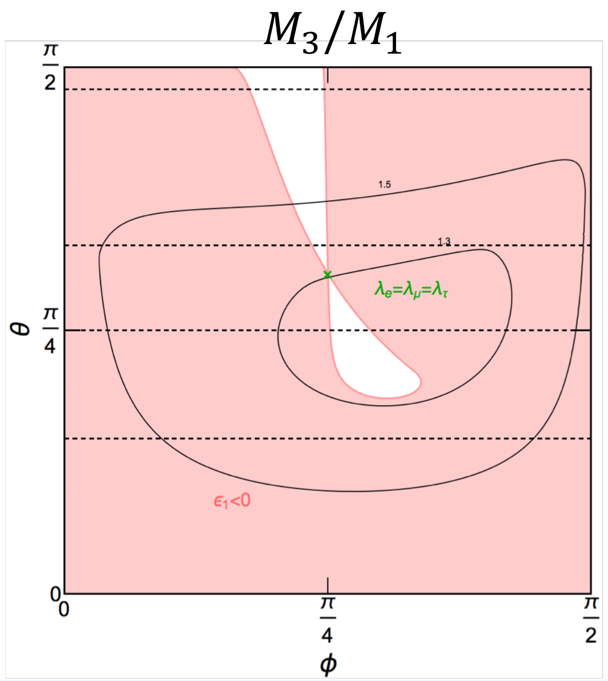
- Minimal gauged $\mathrm{U}(1)_{L_\alpha - L_\beta}$ models are driven into a corner

Leptogenesis

Leptogenesis







$$\epsilon_1 \equiv \frac{\Gamma(N_1 \to H\ell) - \Gamma(N_1 \to H^*\bar{\ell})}{\Gamma(N_1 \to H\ell) + \Gamma(N_1 \to H^*\bar{\ell})}$$

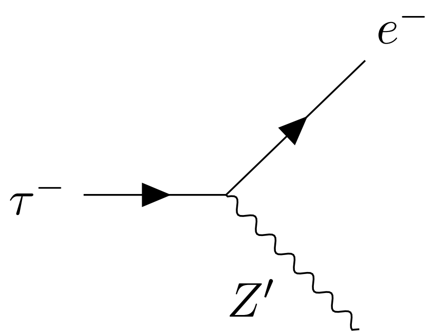
$$Y_{\Delta B} \equiv \frac{n_B}{s} \simeq 8.7 \times 10^{-11} > 0$$

$$\frac{n_B}{n_L} = -\frac{28}{79}$$

$$\epsilon_1 < 0$$

Backup

LFV process



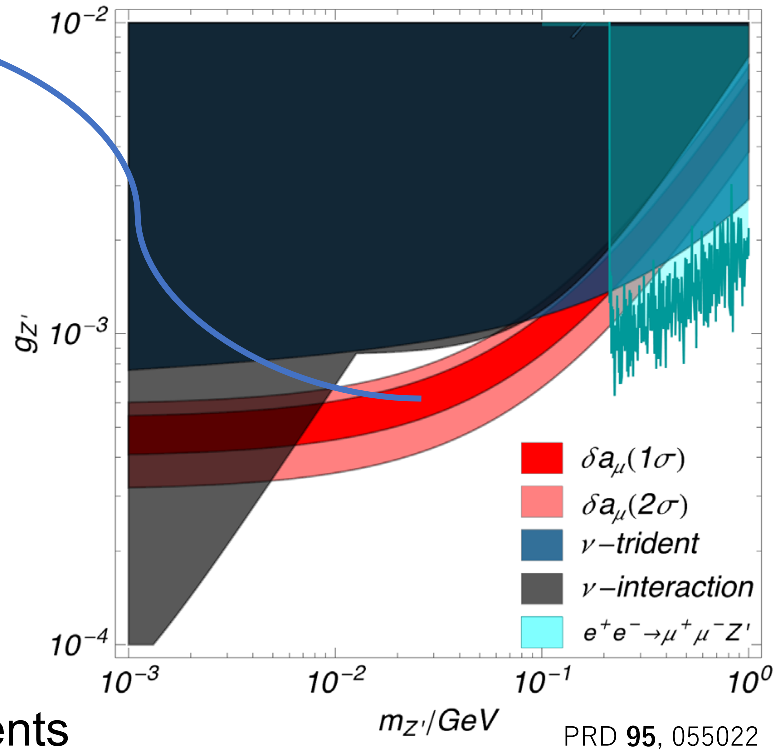
$$10\ \text{MeV} \lesssim m_{Z'} \lesssim 100\ \text{MeV}$$
$$5\times10^{-4} \lesssim g_{Z'} \lesssim 10^{-3}$$

$$\mathcal{L}_{Z'} = g_{Z'}\overline{\ell'}\gamma^\mu \left[U_L^\dagger Q_{\mu-\tau} U_L P_L + U_R^\dagger Q_{\mu-\tau} U_R P_R\right] \ell' Z'_\mu$$

$$\Gamma(\tau\to eZ') = \frac{g_{Z'}^2 m_\tau}{128\pi}\sin^2 2\theta_L \left(2+\frac{m_\tau^2}{m_{Z'}^2}\right)\left(1-\frac{m_{Z'}^2}{m_\tau^2}\right)^2$$

$$\mathrm{BR}(\tau\to eX) \lesssim 2.7\times10^{-3}$$

ARGUS, Z. Phys. C (1995) 68: 25



PRD **95**, 055022

charged lepton-mixing are constrained severely by experiments

LFV process

$$|\sin 2\theta_L| < \begin{cases} 7\times 10^{-5} & \text{for } m_{Z'} = 100\ \text{MeV and } g_{Z'} = 10^{-3} \\ 1\times 10^{-5} & \text{for } m_{Z'} = 10\ \text{MeV and } g_{Z'} = 5\times 10^{-4} \end{cases}$$

$\theta_{L,R} \simeq 0$ or $\underbrace{\pi/2}$

$$U_{L,R} \simeq \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \Rightarrow Q_{\mu-\tau} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \to \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = Q_{\mu-e}$$

$\Rightarrow$ Symmetry group S_3 : $M_l = D_3(g) \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} D_3^T(g)$

$$g = \underbrace{g_{e\mu\tau}\,, g_{e\tau\mu}}_{\mathrm{U}(1)_{L_\mu - L_\tau}}\,, \cancel{\underbrace{g_{\mu e\tau}\,, g_{\mu\tau e}}_{\mathrm{U}(1)_{L_e - L_\tau}}}\,, \cancel{\underbrace{g_{\tau e\mu}\,, g_{\tau\mu e}}_{\mathrm{U}(1)_{L_e - L_\mu}}}$$

$m_{Z'}$ below EW scale
ruled out by experiments

JHEP (2018) 2018: 53
JHEP (2018) 2018: 94
PRL **113**, 091801
…

LFV decay

$$M_\ell = U_L^* \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} U_R^T \qquad U_{L,R} = \begin{pmatrix} \cos\theta_{L,R} & 0 & e^{-i\phi}\sin\theta_{L,R} \\ 0 & 1 & 0 \\ -e^{i\phi}\sin\theta_{L,R} & 0 & \cos\theta_{L,R} \end{pmatrix}$$

$$\mathcal{L}_{Z'} = g_{Z'}\overline{\ell'}\gamma^\mu \left[U_L^\dagger Q_{\mu-\tau} U_L P_L + U_R^\dagger Q_{\mu-\tau} U_R P_R\right] \ell' Z'_\mu \qquad \frac{\tan\theta_R}{\tan\theta_L} = \frac{m_e}{m_\tau}$$

$$Q_{\mu-\tau} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad |y_{e\tau} v_1| = \frac{(m_\tau^2 - m_e^2)\sin 2\theta_L}{\sqrt{(m_\tau^2 + m_e^2) + (m_\tau^2 - m_e^2)\cos 2\theta_L}}$$

$$\Gamma(\tau \to eZ') = \frac{g_{Z'}^2 m_\tau}{32\pi}\left[\left|\left(U_L^\dagger Q_{\mu-\tau} U_L\right)_{13}\right|^2 + \left|\left(U_R^\dagger Q_{\mu-\tau} U_R\right)_{13}\right|^2\right]\left(2 + \frac{m_\tau^2}{m_{Z'}^2}\right)\left(1 - \frac{m_{Z'}^2}{m_\tau^2}\right)^2$$

$$\mathrm{BR}(\tau^- \to e^-\mu^+\mu^-) < 2.7 \times 10^{-8} \qquad \mathrm{BR}(\tau \to e\gamma) < 3.3 \times 10^{-8}$$

PLB 687:139-143,2010 PRL 104:021802,2010

$\mu - e$ mixing

Massless $m_{Z'}$

$\mathrm{BR}(\mu \to eX) \lesssim 10^{-6}$

Phys. Rev. D **34**, 1967

$13\ \mathrm{MeV} \lesssim m_{Z'} \lesssim 80\ \mathrm{MeV}$

$\mathrm{BR}(\mu \to eX) \lesssim 10^{-5}$

Phys. Rev. D **91**, 052020

$m_{Z'} \lesssim 100\ \mathrm{MeV}$

$\mathrm{BR}(\mu \to eX) \lesssim 10^{-4}$

Phys. Rev. Lett. **57**, 2787

Z' contributes to $\mu \to e\gamma$ loop corrections

$\mathrm{BR}(\mu \to e\gamma) \lesssim 10^{-13}$

Eur. Phys. J. C (2016) 76: 434

Analysis of M_ν : Model 1

$$M_D = \frac{v}{\sqrt{2}} \begin{pmatrix} \lambda_e & 0 & 0 \\ 0 & \lambda_\mu & 0 \\ 0 & 0 & \lambda_\tau \end{pmatrix} \qquad M_R = \begin{pmatrix} M_{ee} & \lambda_{e\mu}\langle\sigma\rangle & \lambda_{e\tau}\langle\sigma\rangle \\ \lambda_{e\mu}\langle\sigma\rangle & 0 & M_{\mu\tau} \\ \lambda_{e\tau}\langle\sigma\rangle & M_{\mu\tau} & 0 \end{pmatrix}$$

$$M_\nu = -M_D M_R^{-1} M_D^T$$

$$M_\nu = U_\nu^* M_\nu^{\mathrm{d}} U_\nu^\dagger$$

$$U_{\mathrm{PMNS}} \equiv U_L^\dagger U_\nu$$

$$U_{\mathrm{PMNS}} = \overbrace{\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}}^{V\equiv} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_3}{2}} \end{pmatrix} \equiv U_L^\dagger U_\nu = D_3^T(g) U_\nu$$

Two-zero minor

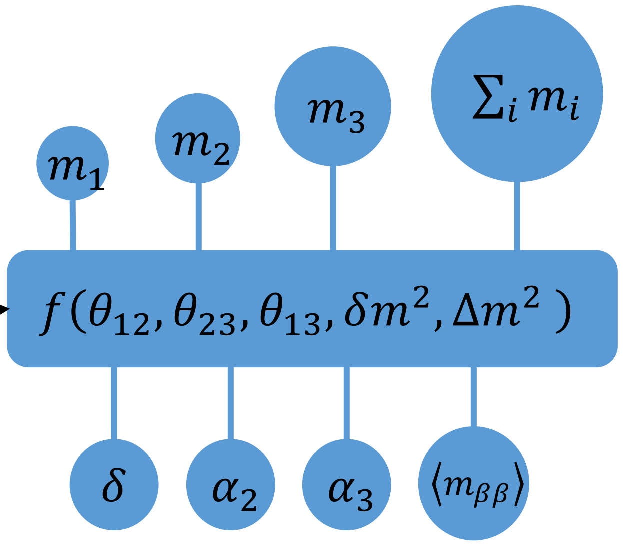
$$\nu_{L\alpha} = \sum_{j=1}^{3} (U_{PMNS})_{\alpha j} \nu_{Lj} \ (\alpha = e, \mu, \tau)$$

$$M_\nu^{-1} = -(M_D^{-1})^T M_R M_D^{-1} = \begin{pmatrix} * & * & * \\ * & \boxed{0} & * \\ * & * & \boxed{0} \end{pmatrix} = U_\nu (M_\nu^{\mathrm{d}})^{-1} U_\nu^T = D_3(g) U_{\mathrm{PMNS}} (M_\nu^{\mathrm{d}})^{-1} U_{\mathrm{PMNS}}^T D_3^T(g)$$

Analysis of M_ν : Model 1

$$\left[D_3(g)U_{\rm PMNS}{\rm diag}(m_1^{-1}, m_2^{-1}, m_3^{-1})U_{\rm PMNS}^T D_3^T(g)\right]_{\mu\mu} = 0$$
$$\left[D_3(g)U_{\rm PMNS}{\rm diag}(m_1^{-1}, m_2^{-1}, m_3^{-1})U_{\rm PMNS}^T D_3^T(g)\right]_{\tau\tau} = 0$$

$$\frac{1}{m_1}V_{\mu 1}^2 + \frac{1}{m_2}V_{\mu 2}^2\,{\rm e}^{i\alpha_2} + \frac{1}{m_3}V_{\mu 3}^2\,{\rm e}^{i\alpha_3} = 0$$
$$\frac{1}{m_1}V_{\tau 1}^2 + \frac{1}{m_2}V_{\tau 2}^2\,{\rm e}^{i\alpha_2} + \frac{1}{m_3}V_{\tau 3}^2\,{\rm e}^{i\alpha_3} = 0$$

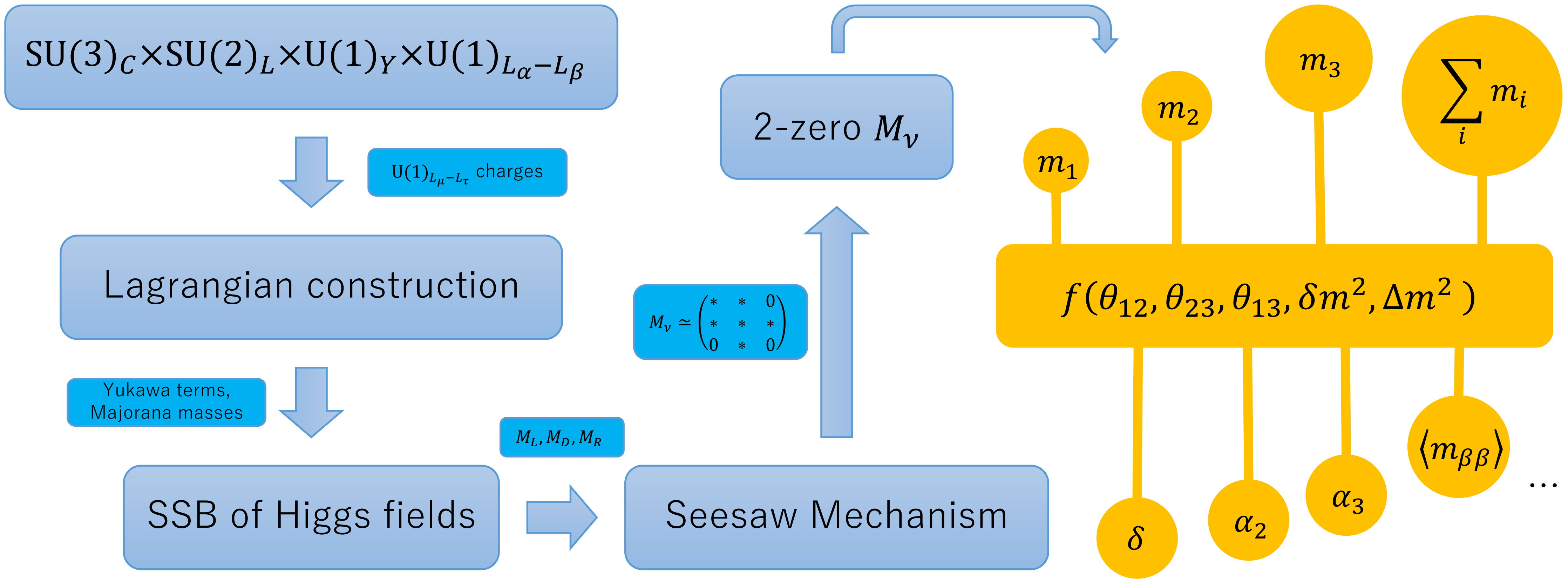


$$\delta m^2 = \Delta m^2_{\rm sol},$$
$$\Delta m^2 = \Delta m^2_{\rm atm} - \frac{1}{2}\Delta m^2_{\rm sol}$$

Analysis of M_ν : Model 2

	$\mathrm{Q}_{\mathrm{L}_\mu-\mathrm{L}_\tau}(\Phi_1)=+1$	$\mathrm{Q}_{\mathrm{L}_\mu-\mathrm{L}_\tau}(\Phi_1)=-1$
M_ν	$-\frac{1}{2}\begin{pmatrix} \frac{v_2^2 y_{11}'^2}{M_{11}} & 0 & v_1 v_2\left(\frac{y_{11}' g_{31}'}{M_{11}}+\frac{y_{33}' g_{12}'}{M_{23}}\right) \\ 0 & 0 & \frac{v_2^2 y_{22}' y_{33}'}{M_{23}} \\ v_1 v_2\left(\frac{y_{11}' g_{31}'}{M_{11}}+\frac{y_{33}' g_{12}'}{M_{23}}\right) & \frac{v_2^2 y_{22}' y_{33}'}{M_{23}} & \frac{v_2^1 g_{31}'^2}{M_{11}} \end{pmatrix}$	$-\frac{1}{2}\begin{pmatrix} \frac{v_2^2 y_{11}'^2}{M_{11}} & v_1 v_2\left(\frac{y_{11}' g_{21}'}{M_{11}}+\frac{y_{22}' g_{13}'}{M_{23}}\right) & 0 \\ v_1 v_2\left(\frac{y_{11}' g_{21}'}{M_{11}}+\frac{y_{22}' g_{13}'}{M_{23}}\right) & \frac{v_1^2 g_{21}'^2}{M_{11}} & \frac{v_2^2 y_{22}' y_{33}'}{M_{23}} \\ 0 & \frac{v_2^2 y_{22}' y_{33}'}{M23} & 0 \end{pmatrix}$

Basic Idea of the analysis



Analysis of M_ν

Scalar-doublet case

$$\begin{cases} e^{-i\alpha_2} = \dfrac{m_1}{m_2} R_2 \\ e^{-i\alpha_3} = \dfrac{m_1}{m_3} R_3 \end{cases}$$

$$\begin{cases} \dfrac{m_2}{m_1} = |R_2| \\ \dfrac{m_3}{m_1} = |R_3| \end{cases}$$

$$\langle m_{\beta\beta} \rangle = \left| \sum_{i=1}^{3} U_{ei}^2 m_i \right|$$

$$\delta$$

$$m_1 = \underbrace{\sqrt{\frac{\delta m^2}{|R_2|^2 - 1}} = \sqrt{\frac{\Delta m^2 + \frac{\delta m^2}{2}}{|R_3|^2 - 1}}}_{\delta}$$

$$\begin{cases} \delta m^2 = m_2^2 - m_1^2 \\ \Delta m^2 = m_3^2 - \dfrac{m_1^2 + m_2^2}{2} \end{cases}$$

$NO: m_1 < m_2 < m_3$

$IO: m_3 < m_1 < m_2$

$$\begin{cases} m_2 = \sqrt{\dfrac{|R_2|^2 \delta m^2}{|R_2|^2 - 1}} \\ m_3 = \sqrt{\dfrac{|R_3|^2 \delta m^2}{|R_2|^2 - 1}} \end{cases}$$

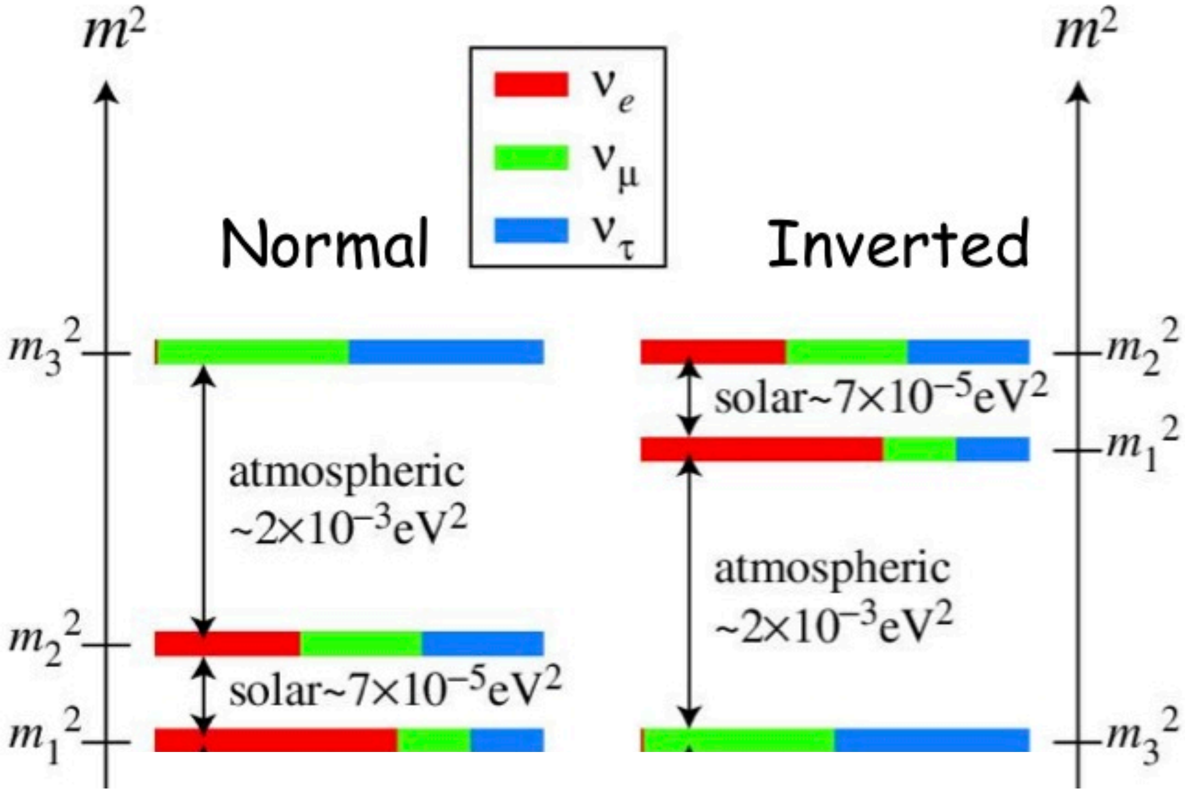
Analysis of M_ν : Model 2 with $\mathrm{U}(1)_{L_\mu - L_\tau}(\Phi_1) = -1$

$$M_\nu = -\begin{pmatrix} \frac{\lambda_e^2 v_2^2}{2M_{ee}} & \frac{\lambda_{e\tau}\lambda_\mu v_1 v_2}{2M_{\mu\tau}} + \frac{\lambda_e \lambda_{\mu e} v_1 v_2}{2M_{ee}} & 0 \\ \frac{\lambda_{e\tau}\lambda_\mu v_1 v_2}{2M_{\mu\tau}} + \frac{\lambda_e \lambda_{\mu e} v_1 v_2}{2M_{ee}} & \frac{\lambda_{\mu e}^2 v_1^2}{2M_{ee}} & \frac{\lambda_\mu \lambda_\tau v_2^2}{2M_{\mu\tau}} \\ 0 & \frac{\lambda_\mu \lambda_\tau v_2^2}{2M_{\mu\tau}} & 0 \end{pmatrix}$$

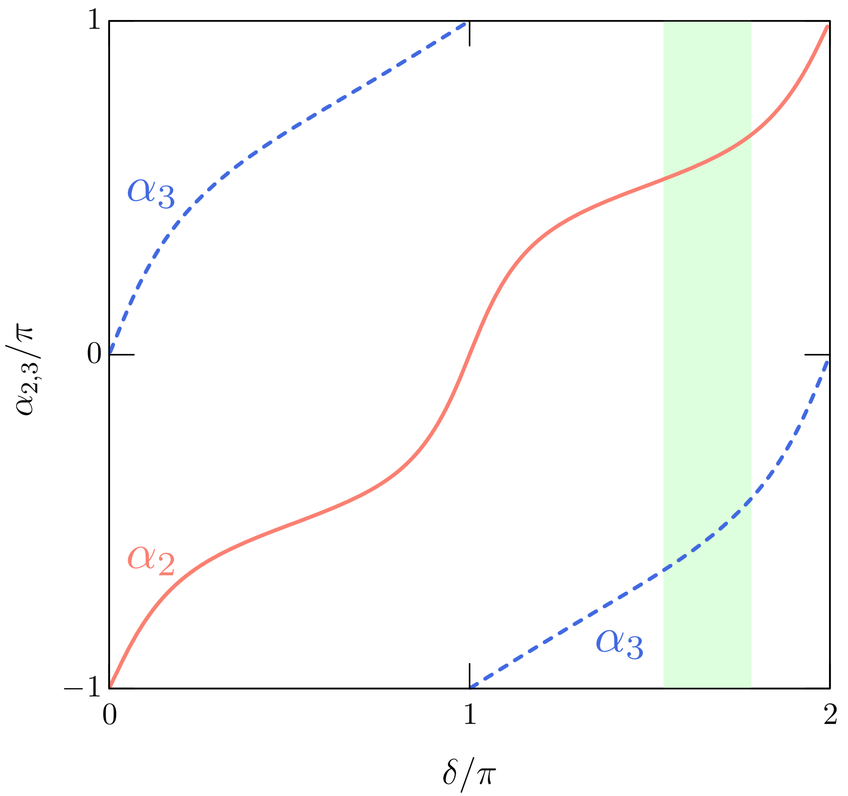
$$\left[D_3(g) U^*_{\mathrm{PMNS}} \mathrm{diag}(m_1, m_2, m_3) U^\dagger_{\mathrm{PMNS}} D_3^T(g)\right]_{e\tau} = 0$$

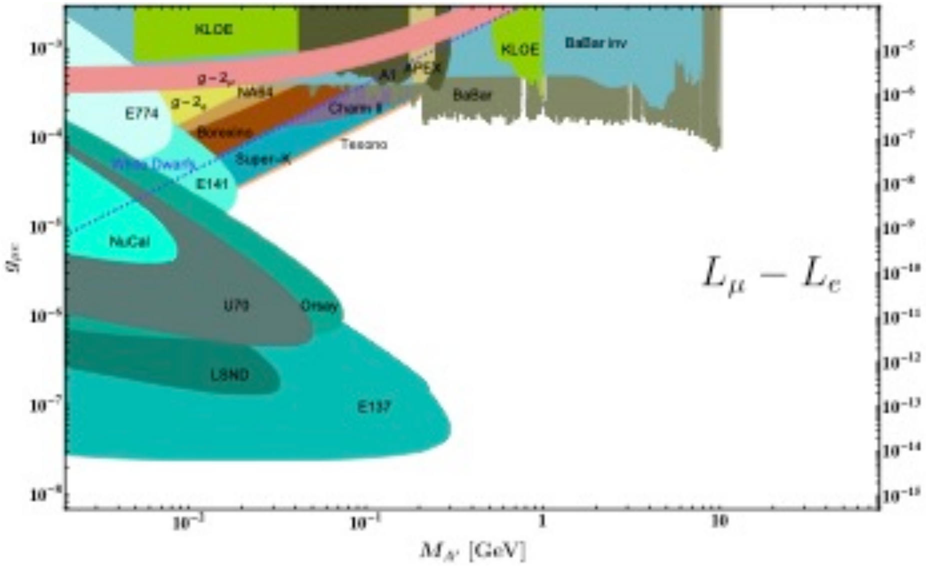
$$\left[D_3(g) U^*_{\mathrm{PMNS}} \mathrm{diag}(m_1, m_2, m_3) U^\dagger_{\mathrm{PMNS}} D_3^T(g)\right]_{\tau\tau} = 0$$

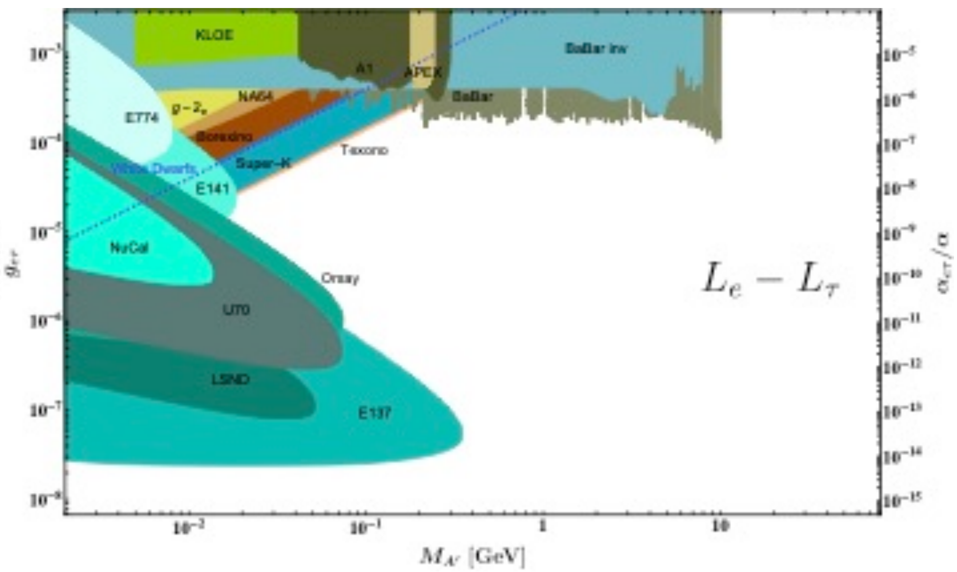
Mass ordering

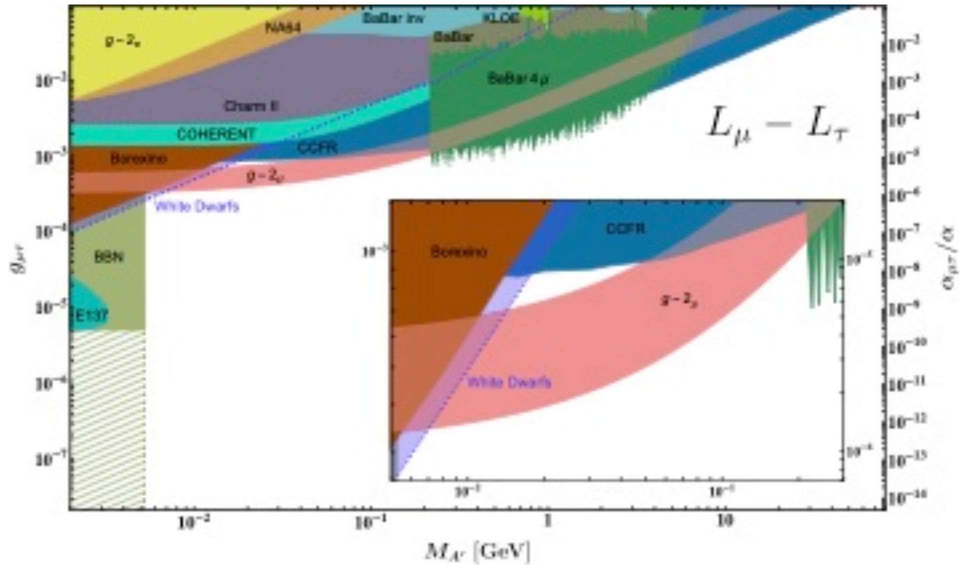


Majorana phase









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